MASS FLOW RATE CHARACTERIZATION OF INJECTORS FOR USE WITH SELF-PRESSURIZING OXIDIZERS IN HYBRID ROCKETS

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ABSTRACT

Interest in self-pressurizing rocket propellants is at an all time high, especially in the field of hybrid propulsion. Due to their high vapor pressure, these propellants can be driven out of a storage tank without the need for complicated pressurization systems or turbopumps, greatly minimizing the overall system complexity and mass. One of the most commonly used self-pressurizing oxidizers for hybrid rockets is nitrous oxide (N$_2$O), which has a vapor pressure of approximately 730 psi at room temperature. However, modeling the feed system can often be challenging due to the presence of two-phase flow, especially in the injector. For this reason, an experimental test apparatus has been developed to study nitrous oxide injector performance over a broad range realistic operating conditions and to validate existing models. Initial testing has characterized the effect of injector geometry and sub-cooling on mass flow rate, with a particular emphasis on the critical flow regime where the flow rate is independent of backpressure. It has been shown that rounded and chamfered inlets provide nearly identical improvement to mass flow rate in comparison to square edged orifices. Due to safety concerns, carbon dioxide (CO$_2$) has been used as an analog to N$_2$O in some of these studies. It is shown experimentally that the critical mass flow rate of carbon dioxide and nitrous oxide are within 10% of each other.

NOMENCLATURE

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INTRODUCTION

Nitrous oxide (N\textsubscript{2}O) has recently been gaining popularity as a liquid oxidizer for use in hybrid rockets, ranging from small scale amateur and academic projects to full scale development programs\textsuperscript{1,2}. Due to its high vapor pressure at moderate temperatures, nitrous oxide is an attractive choice for hybrid rockets because it can be used as a self-pressurizing propellant, eliminating the need for tank pressurization systems or the use of turbopumps. Additionally, it is a safer alternative to more traditional oxidizers owing to its low toxicity, storability, and overall ease of handling. However, to date, little has been published in the open literature on nitrous oxide delivery systems in hybrid rockets with regard to the design and performance of elements such as propellant tanks, feed lines, valves, and injectors.

Specifically, the design of an injector can have a dramatic effect on the overall efficiency and stability of a hybrid rocket system. To demonstrate the importance of injector design, Figure 1 shows the pressure time histories during two hybrid rocket test firings, which are identical in every respect except for the injection schemes. It should be noted that the amplitude of the chamber pressure oscillations in the second test were large enough to shake loose the connections to the data acquisition computer which was adjacent to the test stand, resulting in the truncation of test data. The stark difference in combustion stability between the first and second configuration shows the importance of injector design to overall system performance. A detailed description of the rocket design and setup for these tests can be found in Reference 3.

![Figure 1: Pressure time histories from two nearly identical hybrid rocket motor firings with different injector designs. Combustion chamber geometry and feed system are fixed\textsuperscript{3}.

A properly designed injector should provide the desired oxidizer mass flow rate to the combustion chamber, while sufficiently atomizing the liquid to allow for rapid vaporization of the droplets. Previous work by the authors has studied the atomization process, while the focus of this work is on the characterization of mass flow rate for varying injector designs and operating conditions\textsuperscript{4}. A background of the current methods used to predict mass flow rate through injectors using high vapor pressure propellants is presented next, followed by a description of experimental test campaign that is underway. Finally, some initial results are presented.

BACKGROUND THEORY

INCOMPRESSIBLE LIQUID FLOW

Typical rocket propellants such as liquid oxygen, kerosene and liquid hydrogen are often adequately modeled as incompressible liquids. For incompressible fluid flow through an injector orifice, the well known “$C_d A$” equation for the volumetric flow rate, $Q$, and the mass flow rate, $\dot{m}$, is shown in Eqn. (1):
\[ \dot{m} = Q \rho = C_d A \sqrt{\frac{2 \rho \Delta P}{1 - \left(\frac{A}{A_1}\right)^2}} \]  

where \( C_d \) is the dimensionless discharge coefficient, \( \rho \) is the mass density, \( A \) is the cross-sectional area of the orifice, and \( A_1 \) is the upstream cross-sectional area. The variable \( \Delta P \) is defined as the pressure drop across the injector and is equal to \( P_1 - P_2 \), where \( P_1 \) is the pressure upstream of the injector and \( P_2 \) is the downstream pressure (or backpressure). The denominator under the square root in Eqn. (1) is often wrapped into the discharge coefficient by introducing the meter coefficient, \( C \), as defined in Eqn. (2), and Eqn. (1) is reduced to Eqn. (3):

\[ C = \frac{C_d}{\sqrt{1 - \left(\frac{A}{A_1}\right)^2}} \]  

\[ \dot{m} = CA \sqrt{2 \rho \Delta P} \]  

In many practical cases relating to injectors, \( A \ll A_1 \), and the meter coefficient, \( C \), approaches the discharge coefficient, \( C_d \). For this reason, the “\( C_d A \)” equation is often presented in the form of Sutton and Biblarz as shown in Eqn. (4). In the experimental portion of the current work, Sutton’s form of the mass flow rate equation will be used when referring to the discharge coefficient, \( C_d \).

\[ \dot{m} = C_d A \sqrt{2 \rho \Delta P} \]  

**COMPRESSIBILITY EFFECTS**

The “\( C_d A \)” equation is often used in practice to predict the mass flow rate of traditional liquid propellants through an injector orifice with sufficient accuracy. Unfortunately, this method is often applied inappropriately to injectors of high vapor pressure propellants such as nitrous oxide, with significant error introduced by compressibility effects (compressibility factor \( Z \sim 0.12 \) for saturated liquid, \( Z \sim 0.57 \) saturated vapor at room temperature\(^8\)\(^7\)) as well as the development of two-phase flow within the orifice. To account for compressibility effects, a compressibility correction factor, \( Y \), can be introduced as in Eqn. (5).

\[ \dot{m} = C_d Y A \sqrt{2 \rho \Delta P} \]  

For ideal gases, the compressibility correction factor is well known and can be found in Reference 8. However, for compressible liquids, the correction factor can be calculated in the same fashion as that of Zimmerman et al.\(^9\) In that work, the correction factor was calculated for venturi flow rate measurements of nitrous oxide, which are also governed by Eqn. (5). This form of the compressible liquid correction factor, \( Y' \), is outlined in Eqns. (6) and (7) and is also valid for injectors, but only under the assumption of isentropic flow through the orifice:

\[ Y' = \frac{P_t}{2 \Delta P} \left( \frac{2n}{n-1} \right) \left( \frac{\Delta P}{P_t} \right)^{\frac{2}{n}} \left[ 1 - \left( \frac{\Delta P}{P_t} \right)^{\frac{n-1}{n}} \right] \]  

\[ 0 < \frac{P_t}{2 \Delta P} \left( \frac{2n}{n-1} \right) \left( \frac{\Delta P}{P_t} \right)^{\frac{2}{n}} \left[ 1 - \left( \frac{\Delta P}{P_t} \right)^{\frac{n-1}{n}} \right] < 1 \]
where \( P_t \) is the upstream stagnation pressure, and \( n \) is the isentropic power law exponent as defined in Reference 10. When \( A \ll A_1 \), the upstream pressure \( P_t \) can be used for \( P_1 \). As long as an appropriate method for calculating \( Z \) exists (e.g., highly accurate equations of state exist for nitrous oxide which have been programmed by the National Institute of Standards and Technology in the user friendly REFPROP computer program\(^6,7\)), the above equations can be applied to nitrous oxide as it can be described by the real gas equation of state shown in Eqn. (8)\(^9\).

\[ P = Z \rho RT \]  

(8)

where \( R \) is the specific gas constant. Zimmerman et al. concluded that in flow venturis, where the value of \( \Delta P \) is generally small (< 50 psi), the effects of compressibility can be negligible under certain conditions. However, injectors often establish very large pressure drops (>> 50 psi), and therefore \( Y' \) may deviate significantly from unity. Figure 2 shows the compressible liquid correction factor as calculated for nitrous oxide at an upstream pressure of 1000 psia over a range of temperatures. As the nitrous oxide approaches the critical temperature (557.1 °R) and the compressibility increases, the compressible liquid correction factor becomes more important. At low values of \( \Delta P \), the effect is always small as expected, however as \( \Delta P \) increases, so too does the importance of accounting for compressibility. While the use of a compressibility correction can provide improved accuracy of the “\( C_dA \)” equation for single phase liquid nitrous oxide injectors, of greater concern is the onset of two-phase flow. As will be shown, at the highest \( \Delta P \) values in Figure 2, the effects of two-phase flow within the injector element overshadow the effects of compressibility, thus rendering the classical form of the “\( C_dA \)” equation unsuitable for high vapor pressure propellants such as nitrous oxide.

Figure 2: Compressible liquid correction factor vs. \( \Delta P \) across the injector for \( P_t = 1000 \) psia over a range of nitrous oxide temperatures (\( T_c = 557.1 \) °R). The data used to produce this plot was calculated using REFPROP\(^6,7\).
TWO-PHASE FLOW

Most traditional propellants are used as sub-cooled liquids due to their low vapor pressures and typically high operating pressures. Assuming the discharge coefficient is well characterized for a given injector design, the “\( C_d A \)” equation can be used with a high degree of confidence for these traditional propellants. For self-pressurizing propellants such as nitrous oxide, the operating pressures in the feed system are often very close to the vapor pressure. As a result, local static pressures within the injector can reach values well below the vapor pressure as the liquid accelerates and expands. Under these conditions, cavitation and flash vaporization can occur within the injector orifice, greatly decreasing the bulk fluid density. Figure 3 (adapted from Dyer et al.\(^{11}\)) schematically illustrates the evolution of static pressure throughout an injector for a low vapor pressure propellant in comparison to that of a high vapor pressure propellant.

![Figure 3: Conceptual injector pressure history as adapted from work by Dyer et al.\(^{11}\) showing a low vapor pressure propellant (left) and a high vapor pressure propellant (right) with flow from left to right.](image)

Figure 3(a) shows a typical static pressure profile for a low vapor pressure propellant as it passes through an injector. It should be noted that the vena contracta (v.c.) depicted in Figure 3(a) is caused by flow separation and may or may not contain vapor pockets, depending on the pressure level in the vicinity of the sharp edge. In cases where the bulk static pressure drops below the vapor pressure within the injector as shown in Figure 3(b), significant vapor formation will occur, and the mass flow rate will be limited. This effect was demonstrated by the work of Hesson and Peck, in which the mass flow rate of saturated carbon dioxide (CO\(_2\)) flow through simple thin-plate orifices was studied\(^{12}\). Similar to nitrous oxide, carbon dioxide is a high vapor pressure liquid at room temperature. In fact, often carbon dioxide is used as an analog to nitrous oxide because the thermodynamic and transport properties of the two are so similar (this will be discussed later in detail). Figure 4 shows mass flow rate versus downstream pressure, \( P_2 \), for lines of constant upstream pressure \( P_1 \) as presented by Hesson and Peck. This figure shows that for a given upstream pressure, a maximum mass flow rate is achieved as the downstream pressure drops below some critical value, essentially choking the orifice with regard to mass flow. This is referred to as critical flow, and in this regime the mass flow rate is dependent upon only the upstream conditions. In order to accurately model the mass flow rate in two-phase injectors, this critical flow phenomenon must be accounted for.
A model that is commonly used to predict the two-phase critical flow rate through an injector orifice is called the Homogeneous Equilibrium Model (HEM). In this model, it is assumed that the liquid and vapor phases are in thermal equilibrium, there is no velocity difference between the phases, and the flow is isentropic across the injector. The critical mass flow rate is calculated by seeking a maximum in the mass flow rate expression derived from the first law of thermodynamics which is shown in Eqn. (9) and by following a line of constant entropy.

\[ \dot{m}_{\text{HEM}} = A\rho_v \sqrt{2(h_1 - h_2)} \]  
\[ s_1 = s_2 \]  

Here, \( h_1 \) is the upstream stagnation specific enthalpy, \( h_2 \) is the downstream specific enthalpy, \( s_1 \) is the upstream stagnation specific entropy, and \( s_2 \) is the downstream specific entropy. Again, when \( A \ll A_1 \), \( h_1 \) is approximately equal to \( h_1^* \), the static specific enthalpy just upstream of the injector and \( s_1 \) can be approximated as \( s_1^* \), the static specific entropy just upstream of the injector. The resulting equations which can be used to predict the mass flow rate are Eqns. (11) and (12).

\[ \dot{m}_{\text{HEM}} = A\rho_v \sqrt{2(h_1 - h_2)} \]  
\[ s_1 = s_2 \]  

When the upstream temperature and pressure are known, as well as the backpressure, the calculation of mass flow rate for nitrous oxide using the Homogeneous Equilibrium Model is made trivial through the use of highly accurate equations of state programmed in REFPROP. At critical flow, the HEM mass flow exhibits a maximum with respect to the downstream pressure as described in Eqn. (13).

\[ \dot{m}_{\text{Crit,HEM}} = \dot{m}_{\text{HEM}} \left| \left. \frac{\partial \dot{m}_{\text{HEM}}}{\partial p_2} \right|_{p_2=0} \right. \]  

A graphical representation of this type of critical flow calculation is included in Figure 5, which models an injector for saturated liquid nitrous oxide at room temperature (528 °R). It is clear from the plot that for downstream pressures below the critical value, the equilibrium model does not level off in the same way seen in the experimental data, such as that of Hesson and Peck. Therefore while it may be appropriate to use the HEM as a means to predict the critical mass flow (subject to validation), it is not necessarily suitable for calculations of the mass flow rate in general at pressure drops greater than the critical value.
Figure 5: Critical mass flow rate prediction using the Homogeneous Equilibrium Model for initially saturated liquid nitrous oxide (736 psia) at room temperature (528 °R). The data used to produce this plot was calculated using REFPROP.$^6$,$^7$

It is expected that when the assumption of equilibrium is valid, the HEM should do a good job of predicting the critical mass flow rate. However, thermodynamic equilibrium is only a limiting case, and gives a lower-bound estimate for the critical mass flow rate.$^{15}$ Furthermore, experimental data suggests that the critical mass flow rates for many short tube injectors are significantly underpredicted using the HEM, likely due to non-equilibrium effects.$^{11}$

TWO-PHASE FLOW WITH NON-EQUILIBRIUM EFFECTS

The equilibrium model described above begins to break down when the finite rate of heat and mass transfer between the liquid and vapor phases prohibits the injector flow from reaching thermodynamic equilibrium. Dyer et al. proposed that non-equilibrium effects were predominantly cause by two processes: superheating of the liquid during expansion and finite vapor bubble growth rates.$^{11}$

In hybrid rocket feed systems, nitrous oxide is either used as a saturated liquid, or a secondary pressurization system is used to sub-cool the liquid in a process called supercharging. In both cases, the static pressure drops as it accelerates and expands through the injector. When the pressure falls below the saturation pressure, vapor formation may initiate almost immediately, especially when nucleation sites are readily available. However, often this is not the case, and instead the liquid enters a metastable state.$^{11}$ In this scenario, the nitrous oxide becomes a superheated liquid, at which point very small disturbances or the introduction of nucleation sites can result in local flashing of the liquid to vapor.

The extent to which the liquid can theoretically be superheated in the metastable state is bounded by the liquid spinodal line. The spinodal lines mark the limits of the region where only unstable fluid can exist, therefore homogeneous nucleation must occur at some point before reaching the spinodal line.$^{16}$ Thus, the metastable region is located between the saturation line and the spinodal line, which is depicted graphically by the purple region in Figure 6. The spinodal lines can be calculated by following the traverse of the outermost isothermal local extrema (i.e. the extrema closest to the saturation curves). Specifically, the liquid and vapor spinodal lines are collocated with the outermost isothermal local minima and maxima respectively, and they meet at the thermodynamic critical point. The ability of the metastable liquid to approach the spinodal
limit is dependent upon the rate of depressurization and the level of disturbances that would tend to cause heterogeneous nucleation and flashing to vapor\textsuperscript{16,17}. Depending on the rate of depressurization, the liquid follows a process bounded by the isentropic and isothermal curves as shown in Figure 6. The isentropic path corresponds to an infinitely fast depressurization and the isothermal line corresponds to an extremely slow process, whereas an actual fluid trajectory will lie somewhere between the two\textsuperscript{17}. Due to the complexities in determining how far into the metastable region the fluid travels and the actual thermodynamic path that it takes, some models do not attempt to resolve this trajectory, but find other ways to account for the non-equilibrium effects\textsuperscript{11}.

Figure 6: $P$-$\rho$ Diagram for nitrous oxide. Lines of constant temperature (550 °R) and constant entropy are included for a sub-cooled liquid which starts at 1000 psia and transitions into the metastable region. The data used to produce this plot was calculated using REFPROP\textsuperscript{6,7}.

In order to account for the finite rate of vapor bubble growth, Dyer et al. defined a characteristic bubble growth time as shown in Eqn. (14).

$$\tau_b \equiv \frac{3}{2} \frac{\rho_l}{P_v - P_2}$$

(14)

The amount of vaporization that can occur within an injector element is dependent upon the ratio of the bubble growth time to the fluid residence time, which is inversely proportional to the velocity of the flow as shown in Eqn. (15). By comparing the bubble growth time with the liquid residence time, Dyer et al. introduced a non-equilibrium parameter, $k$, which is shown in Eqn. (16).

$$\tau_r \equiv \frac{L}{u} = \frac{L}{\sqrt{2\Delta P}} = \frac{\rho_l}{\sqrt{2\Delta P}}$$

(15)

$$k = \frac{P_1 - P_2}{P_v - P_2} \propto \frac{\tau_b}{\tau_r}$$

(16)

A model first proposed by Dyer et al. and later corrected by Solomon accounts for non-equilibrium effects by allowing the flow rate to vary smoothly between that predicted by the
Homogeneous Equilibrium Model and that of the incompressible “$C_d A$” equation\(^8\). The form of this model is shown in Eqn. (17) and is referred to by Solomon as the Non-Homogeneous Non-Equilibrium (NHNE) model.

$$\dot{m}_{NHNE} = C_d A \left(1 - \frac{1}{1 + \kappa}\right)\dot{m}_{SPI} + \frac{1}{1 + \kappa} \dot{m}_{HEM}$$

(17)

This model attempts to account for the fact that when the residence time of bubbles $\tau_r$ is much less than the characteristic time for bubble growth $\tau_b$, very little vapor will be formed by the exit, and the single phase assumption is likely valid. However, when $\tau_b << \tau_r$, it is proposed that the flow rate should approach the critical value predicted by the Homogeneous Equilibrium Model. It has been observed that this is indeed the case for injectors with large length to diameter ratio $L/D$, which is attributed to the fact that there is more time for the fluid to reach thermodynamic equilibrium\(^13\).

It should be noted that as the downstream pressure rises above the vapor pressure, the non-equilibrium parameter becomes complex due to the negative values under the radical. This is not a problem for systems using saturated nitrous oxide because the backpressure rarely approaches $P_v$, but this deficiency must be addressed for systems using initially sub-cooled liquid. The NHNE model has been compared to a limited set of experimental hot fire test data at Stanford University and has demonstrated reasonable agreement. A constant $C_d$ of 0.66 was used, based on the average value for all of the injectors that were included. Even with an averaged discharge coefficient value, the model predicts mass flow rates with accuracy to within $\pm 15\%$ for most tests\(^11\). However, better predictions are desired to assist in the development of full scale hybrid rockets using nitrous oxide.

Each of the models outlined above are conditionally useful, but a common shortcoming is the requirement for a well characterized injector discharge coefficient. While the characterization of the discharge coefficient for most traditional rocket propellant injectors is well understood, there is a great deal of uncertainty when it comes to the design and performance of nitrous oxide injectors. Additionally, there has been little in the way of model comparison and validation for injectors operating at realistic operating conditions with high vapor pressure propellants. While it is standard practice in the development of hybrid rockets to perform cold flow testing (oxidizer flow only, no ignition) in order to characterize a given injector design in terms of mass flow rate and atomization, most cold flow tests are not performed at the actual operating backpressure observed during combustion testing. Often cold flow tests are performed with the combustion chamber removed and the injector flowing to ambient pressure, resulting in a much larger $\Delta P$ than would be expected during tests involving combustion for the same hardware. As should be clear from the theoretical treatment above, the downstream pressure, $P_2$, and the pressure drop, $\Delta P$ are important parameters affecting the actual performance and modeling of injector mass flow.

The focus of this work now shifts to describe a new experimental facility for the cold flow testing of nitrous oxide injectors at realistic operating pressures. The goal of this testing is to record and compile injector performance data over a broad range of operating conditions, which can be used to compare the accuracy of different mass flow rate prediction techniques described above and elsewhere in the literature. The experimental results from this apparatus will also be used to support the design efforts of hybrid rocket development programs at Stanford University and NASA Ames Research Center. Some initial results are presented for five injector designs.

**EXPERIMENTAL SETUP**

In order to effectively characterize the mass flow rate performance of a variety of injector designs, a new facility has been developed to allow for the establishment of realistic operating pressures upstream and downstream of an interchangeable injector plate. Transparent polycarbonate tubes are closed on each end by aluminum end caps to create a pressure vessel with optical access (ID = 1”). This transparent setup was chosen to facilitate an ongoing study of
nitrous oxide atomization using high-speed photography and laser illumination as described in Reference 4, the results of which will not be presented here. The optical access does allow for observations of the visible flow conditions upstream and downstream of the injector, which can be particularly useful when it comes to troubleshooting. A cut-away model of the test section is shown in Figure 7 along with a photo of the assembled hardware. Pressure and temperature are measured in both the upstream and downstream polycarbonate chambers at sampling rates from 500 Hz up to 5 kHz, allowing for the instantaneous measurement of $\Delta P$ and determination of the upstream thermodynamic state.

![Figure 7: (a) Cut away view of the test section with main components labeled (b) Photo of the assembled hardware and instrumentation (including 2 Omega K Type thermocouples and 2 Measurement Specialties Inc. MS100 series pressure transducers).](image)

Liquid is supplied to the test section by a separate transparent pressure vessel which is also used for experiments investigating expulsion dynamics for self-pressurized propellant tanks. The tank is shown in Figure 8(a) and serves as an intermediate run tank between the test section and the large industrial compressed gas cylinders (K size) that the nitrous oxide is delivered and stored in. More information on the tank experiments can be found in the work of Zimmerman et al. Liquid flows from the intermediate run tank to the test section through $\frac{1}{4}$" tubing, a pneumatically actuated ball valve, and a venturi flow meter. A standard pressure transducer and a differential pressure transducer are used in conjunction with the venturi to calculate mass flow rate. A photo of the full system setup is shown in Figure 8(b).
The run tank is filled using a pneumatically driven pump, but under certain conditions the self-pressurization within the storage tank is sufficient to drive the nitrous oxide into the run tank. Many practical systems employ supercharging in order to avoid cavitation in the feed system caused by localized pressure drops. In order to sub-cool the liquid in this fashion, the run tank can be pressurized with helium up to approximately 1000 psig. The pressurization system allows for flexibility when it comes to setting the thermodynamic state of nitrous oxide in the tank and upstream of the injector. Helium is widely chosen as the supercharging gas in practical pressurization systems due to its low molecular weight. However, if helium becomes dissolved in a self-pressurizing propellant there can be significant effects on cavitation due to helium desorption throughout feed system. In current tests, it is believed that helium is not appreciably dissolved in the nitrous oxide because of the short time between supercharging and completion of the test. Nevertheless, future work will address the potentially undesirable effects of helium absorption and desorption with regard to cavitation and mass flow rate. A detailed process and instrumentation diagram (P&ID) with more details on the fluid handling and instrumentation of the system can be found in Reference 4.

In order to allow for the testing of a wide range of injector designs, an injector insert was designed to be easily interchangeable. The brass insert is mounted within an aluminum block that serves as the junction between the upstream and downstream polycarbonate pressure vessels as shown above in Figure 7(a). Silicone O-rings are used to provide seals at each junction. A large assortment of injector inserts have been manufactured for testing in this facility. The first injectors to be tested in this rig were designed to investigate the mass flow rate characteristics with the variation of injector orifice diameter $D$ and hole inlet geometry (square edged, rounded, chamfered). For this work, the length $L$ of each injector hole was fixed at a value of 0.73”. However, future work will study the effect of varying overall injector length. All of the injectors used in this round of testing are simple straight hole orifices. Table 1 details the geometry of each injector tested in this work. Rounding on the injector inlets was at a radius of 0.02” and chamfers were machined at 45° to a depth of 0.03”. A photo of one of these injector inserts is shown in Figure 9.

Figure 8: Photos of (a) the intermediate run tank and (b) the overall system setup.
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<td>(3)</td>
<td>Straight</td>
<td>0.059</td>
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<td>0.076</td>
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<td>9.5</td>
<td>Square Edge</td>
</tr>
</tbody>
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Figure 9: Photo of injector insert (3) with corner O-ring installed.

CARBON DIOXIDE AS AN ANALOG TO NITROUS OXIDE

Although nitrous oxide has properties that make it an attractive oxidizer choice for hybrid rockets, it is still a reasonably strong oxidizer and its use does pose some significant risks. Specifically, nitrous oxide has a positive heat of formation, and the potential exists for rapid exothermic decomposition. This type of runaway decomposition reaction can lead to violent and dangerous pressure vessel explosions. Multiple groups have studied these explosive events and have concluded that despite the potential hazards, nitrous oxide can be used safely with proper handling considerations\textsuperscript{2,21}. Stringent cleaning procedures such as those used for oxygen systems must be followed in order to minimize the possibility of explosions. However, these types of explosions do happen in the hybrid rocket industry, and have caused major hardware damage and even fatalities. Therefore, the use of nitrous oxide is not ideal in an academic setting, and is often restricted to certain areas as a means for ensuring student safety.

Fortunately, multiple groups have recognized that carbon dioxide can be used as a safe analog to nitrous oxide when it comes to fluid flow studies\textsuperscript{9,18}. Nitrous oxide and carbon dioxide are quite similar thermodynamically in that they have almost the same molecular weight and exhibit less than 5% variation in most of the important thermodynamic properties listed in Table 2. However, some thermodynamic parameters do differ greatly, such as the triple point and the acentric factor. The acentric factor is a measure of the aspherical nature of a molecule, and its tendency to behave differently than simple monatomic fluids. The difference in acentric factor between nitrous oxide and carbon dioxide can contribute to the difference in properties such as saturated vapor pressure and vapor density\textsuperscript{24}. As seen in Figure 10, these two values can differ upwards of 15% when nearing the critical point (though saturated liquid density matches quite closely). An in depth comparison of nitrous oxide and carbon dioxide thermodynamic and transport properties away from the saturation line is performed by Zimmerman et al. and has been applied to look at the effect of using carbon dioxide as an analog during propellant tank blow-down experiments\textsuperscript{19}. 

\textsuperscript{2}
Table 2: Comparison of thermodynamic properties of N\textsubscript{2}O and CO\textsubscript{2}.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Nitrous Oxide (N\textsubscript{2}O)</th>
<th>Carbon Dioxide (CO\textsubscript{2})</th>
<th>Difference (%)</th>
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<tbody>
<tr>
<td>(M)</td>
<td>lb\textsubscript{m}/lbmol</td>
<td>44.013</td>
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<td>psia</td>
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<td>1070.0</td>
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<tr>
<td>(P_{trip})</td>
<td>psia</td>
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<td>75.1</td>
<td>+491.3</td>
</tr>
<tr>
<td>(T_c)</td>
<td>°R</td>
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<td>547.4</td>
<td>-1.7</td>
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<tr>
<td>(T_{trip})</td>
<td>°R</td>
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<td>+18.8</td>
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</table>

Figure 10: Percent difference in saturated vapor density, liquid density, and vapor pressure (\(P_v\)) for carbon dioxide as an analog to nitrous oxide\textsuperscript{6,7,22,23}. Oxidizer temperatures ranging from 450 °R to 520 °R are of particular importance for hybrid rocket system testing using nitrous oxide.

While the thermodynamic similarities are encouraging, experimental comparisons must be performed to quantify the effects of using carbon dioxide as an analog to nitrous oxide before work can be moved solely to carbon dioxide. This work will provide some initial comparisons of injector performance with the use of carbon dioxide versus nitrous oxide. To accommodate testing with nitrous oxide while ensuring the safety of all involved, this work was performed at the Outdoor Aerodynamic Research Facility (OARF) at NASA Ames Research Center in Moffett Field, CA. Experiments are performed under remote control with test personnel in a bunker approximately 150 ft away from the test stand.

**TEST METHOD AND DATA PROCESSING**

The procedure for tests involving nitrous oxide is described below. However, the same process is used for carbon dioxide, and the name of the fluid can be substituted at will. The only significant difference in operating with nitrous oxide is the strict cleaning and anti-contamination procedures that are used when handling hardware (not described here).
1) Test Preparation

A typical test is prepared by filling the run tank with nitrous oxide to a desired level using the pneumatically driven pump. If it is necessary to establish a desired liquid temperature, the nitrous oxide storage cylinder can be either heated or cooled using an insulated water bath prior to and throughout the test. Next, the helium pressurization system is used to provide the target supercharge level. It is useful at this point to define a local supercharge pressure, which is defined as the difference between the instantaneous static pressure and the instantaneous vapor pressure of pure nitrous oxide at a given point in the system. The expression for the supercharge pressure at in the pre-injector volume located just upstream of the injector is shown in Eqn. (18), where the vapor pressure is evaluated at the local instantaneous temperature ($T_1$ in this case).

$$P_{super}^1 = P_1 - P_v\bigg|_{T=T_1}$$  \hspace{1cm} (18)

Aside from an exhaust valve that opens to ambient, the downstream chamber labeled in Figure 7(a) is a closed vessel. The exhaust valve is generally kept closed throughout the duration of a test and if desired, the downstream chamber and pre-injector volume can be pre-pressurized with nitrogen to establish elevated backpressures at the beginning of a test.

2) Running a Test

Once the desired tank and chamber conditions are established, the pneumatically actuated ball valve that connects the run tank to the test section is opened to initiate a test. Nitrous oxide immediately begins to fill the pre-injector volume, and $P_1$ rises rapidly to a level approximately equal to that of the run tank. At the same time, nitrous oxide is flowing through the injector and the downstream pressure $P_2$ starts to rise. Because $P_1$ increases much faster than $P_2$, a large pressure drop $\Delta P$ is established across the injector (near 1000 psi in some instances). However, because the exhaust valve is closed and the chamber is filling up with liquid nitrous oxide, the downstream pressure will continue to rise until $P_1 = P_2$, at which point the pressure drop across the injector is zero and flow stops, completing the test. By operating in this dynamic fashion, a single test can provide a broad sweep of downstream operating pressures for constant upstream conditions. A typical pressure time history is shown in Figure 11 (valve opens at approximately 0.5 s).

![Figure 11: Pressure time histories for positions upstream and downstream of the injector during a typical test.](image-url)
Once the ball valve is opened, there is a rush of nitrous oxide flow into the pre-injector volume. As the liquid front of nitrous oxide expands rapidly into the pre-injector volume, much of it flashes to vapor, resulting in rapid cooling and low temperatures for a short duration in the beginning of a test as shown in Figure 12. However, once the ensuing liquid fills in the volume upstream of the injector, the temperature returns to the bulk liquid temperature and stabilizes. Because the pressure transducers used in this region are susceptible to rapid changes in temperature, it is important to provide thermal insulation to ensure accurate pressure measurements. Synthetic lubricant (Krytox) is applied to the pressure transducer cavities in order to provide thermal insulation without impacting the accuracy of the measurements.

![Figure 12: Typical time history of \( T_1 \) (minimum temperature of 445 °R).](image)

3) Data Analysis

While the pre-injector volume is filling with liquid nitrous oxide there is no significant flow limiting and a high rate of mass flow is fed through the system. These high mass flow rates during the beginning of a test are reflected in the venturi flow rate measurements as seen in Figure 13(a) from approximately 0.5 s until 0.6 s. Not until this volume is completely filled with liquid does the injector become the mass flow limiting device, and thus the venturi mass flow rate measurements can only be used from this point onwards during a test. For this reason, the test data used for analysis must be selected appropriately. A typical cropping of the data is shown by the highlighted segments in Figure 13(a) and Figure 13(b) which correspond to the same test as Figure 12. The test depicted in these figures is for nitrous oxide flow through injector number 3.

![Figure 13: (a) Mass flow rate time history and (b) pressure time history for a typical test of nitrous oxide flowing through injector number 3.](image)
Looking carefully at the plots in Figure 13, it can be seen that in the early portions of the cropped section, \( \Delta P \) is dropping rapidly while the mass flow remains relatively constant. When the mass flow data is plotted versus the pressure time history, it is much easier to examine the behavior of the injector. This type of analysis results in a plot of the form shown in Figure 14, created from the cropped sections above. This data corresponds to average upstream values of \( P_1 = 704 \) psig, \( T_1 = 505 \) °R, and \( P_{\text{super}} = 169 \) psi. While the instantaneous values for the upstream parameters do shift a small amount during a test, for simplicity the injector performance will be studied based on the average upstream values and the instantaneous downstream values. In a similar fashion it is possible to plot the effective discharge coefficient versus \( \Delta P \) as shown in Figure 15. This effective discharge coefficient is defined by dividing the instantaneous mass flow rate during the test by the value calculated assuming single phase incompressible flow and a discharge coefficient of 1. As \( \Delta P \) and mass flow rate approach zero, the effective discharge coefficient shows a great deal of scatter, and this region is ignored for the purposes of our analysis.

![Figure 14: Nitrous oxide mass flow rate vs. injector \( \Delta P \) data for a test of injector number 3 at average upstream values of \( P_1 = 704 \) psig, \( T_1 = 505 \) °R and \( P_{\text{super}} = 169 \) psi.](image)

![Figure 15: Effective discharge coefficient vs. injector \( \Delta P \) data for a test of injector number 3 at average upstream values of \( P_1 = 704 \) psig, \( T_1 = 505 \) °R and \( P_{\text{super}} = 169 \) psi.](image)
The plots above are useful in identifying the different flow regimes. For low values of \( \Delta P \) (30 psi < \( \Delta P \) < 200 psi in this case), the flow is well described by the SPI model. The experimental data in this region can be fit to Eqn. (4) by determining the SPI discharge coefficient for this combination of injector geometry and upstream thermodynamic conditions \( (c_d \approx 0.77 \text{ in this case}) \). The red line in Figure 14 corresponds to the mass flow rate predicted by the SPI model using this discharge coefficient. While the SPI model is not physically valid in the two-phase flow region past the critical \( \Delta P \), the results from Figure 15 allow one to calculate an effective discharge coefficient for use in the two-phase region. For example, the effective discharge coefficient for injector number 3 at the upstream conditions presented above drops to a value of approximately 0.6 at a pressure drop of 330 psi. For large values of \( \Delta P \), it is clear that we do in fact see the same critical flow effect as shown by Hesson and Peck and predicted by the HEM and NHNE models. As seen in the figures above, a region is identified indicating the transition from the single phase mode into the multiphase regime. The limits of this region are based on the deviation of discharge coefficient away from the calculated SPI value and are defined below:

**Transition \( \Delta P \):** Value of \( \Delta P \) for which the effective discharge coefficient first drops below 99% of the SPI value.

**Critical \( \Delta P \):** Value of \( \Delta P \) for which the effective discharge coefficient first drops below 95% of the SPI value.

The plots above can be used directly to assist in the design of injectors, however only for the specific upstream pressure and supercharge used in this test and with a rounded inlet injector. This type of test and data processing can be repeated for a multitude of upstream conditions and injector configurations in order to create a catalog of performance data for use in the design of injectors, or for validation of predictive models. The rest of this work presents the initial results that are to be included in the injector testing database, along with some basic comparisons.

**RESULTS AND DISCUSSION**

To date over 500 tests have been performed in this new facility, using both nitrous oxide and carbon dioxide as the test fluid. Experiments have been performed over a wide range of operating conditions:

- \( P_1 \): 450 psig \( \rightarrow \) 1000 psig
- \( P_2 \): 0 psig \( \rightarrow \) 1000 psig
- \( T_1 \): 480 °R \( \rightarrow \) 536 °R
- \( P_{\text{super}}^1 \): 0 psi \( \rightarrow \) 500 psi

As described in the previous section, a single test provides a sweep of downstream conditions and a vast amount of injector performance data. In order to cover as much of the design space as possible, a range of tests at different supercharge pressures are performed for a given injector design. Figure 16 provides the resulting experimental nitrous oxide mass flow rate data for injector number 3 at a variety of supercharges, plotted against \( \Delta P \). This figure is created by combining a number of plots similar to that shown in Figure 14, and the same data processing is performed for each curve (the labeling of flow transition and critical mass flow rate however are not included in this plot). Starting at \( \Delta P = 0 \) and moving up and to the right, it is evident that at low values of \( \Delta P \), each test follows the single phase behavior, and the mass flow rate is relatively independent of supercharge. However, continuing up the curve, we encounter the critical \( \Delta P \) corresponding to the lowest supercharge test (in this case \( P_{\text{super}}^1 = 41 \text{ psi} \)), after which point the mass flow rate deviates from the single phase curve, and critical flow is encountered. This process continues while moving up the curve, and the results from each test deviate from the
single phase behavior in order from lowest to highest supercharge. It becomes clear that as $P_{\text{Super}}$ increases, so too does the critical mass flow rate and critical $\Delta P$. From the same test data the effective discharge coefficient is calculated and is plotted versus supercharge and $\Delta P$ as shown in Figure 17. The behavior of the $C_d$ curves can be examined in the same fashion, following the single phase behavior until it deviates at the critical $\Delta P$ value.

![Figure 16: Mass flow rate vs. $\Delta P$ and supercharge for injector number 3 with nitrous oxide.](image)

![Figure 17: $C_d$ vs. $\Delta P$ and supercharge for injector number 3 with nitrous oxide.](image)

At this point it should be noted that all tests reported in this work are for non-zero values of supercharge. There are two main reasons for this: (a) For tests with saturated liquid, large amounts of vapor are formed within the run tank, leading to unreliable venturi mass flow rate measurements, and (b) with only pressure and temperature measurements upstream of the injector, it is not possible to determine the thermodynamic state once entering the two phase region (in other words, for a given temperature and pressure the vapor mass fraction can vary from 0 to 1.). The minimum supercharge used for this work is approximately 41 psi.
The same plots presented above can be created for each combination of injector and fluid. For reference information, all of the data produced during experiments for this work are included in Appendix A. In order to present some of the results, we will (a) first look at some tests using nitrous oxide to investigate the effect of injector hole diameter $D$, (b) examine the validity of using carbon dioxide as an analog to nitrous oxide in order to (c) use the results from tests with carbon dioxide to examine the effect of injector inlet geometry. Finally, the experimental data will be compared to the results from NHNE model presented earlier in this work.

**EFFECT OF INJECTOR HOLE DIAMETER**

Figure 18 shows the SPI discharge coefficient over a range of supercharge values for three different injectors, all square edged orifices with varying diameters. While there is virtually no dependence on supercharge, it can be seen that $C_d$ decreases with increasing hole diameter. It cannot be determined from these tests alone whether the effect is due to the difference in hole diameter or the difference in hole length to diameter ratio, $L/D$ (future work will address this subject). All of these tests were performed using nitrous oxide.

![Figure 18: $C_d$ in the single phase region vs. supercharge for injectors number 1, 2 and 5 with nitrous oxide.](image)

In order to compare mass flow rate performance between injectors of different sizes, it is necessary to introduce the critical mass flux, $G_{crit}$ which is equal to the critical mass flow rate divided by the cross sectional area of the injector hole, as shown in Eqn. (19).

$$ G_{crit} = \frac{m_{crit}}{A} $$  \hspace{1cm} (19) 

Figure 19 shows the critical mass flux for the same three injectors plotted versus supercharge. From the plots, it can be seen that for low values of supercharge, the injector hole diameter does have some effect on the critical mass flux, however at higher values of supercharge, the effect of hole diameter is much greater. When Figure 18 and Figure 19 are compared, it is interesting to note that the order of increasing critical mass flux does not match the order of increasing SPI discharge coefficient. Also, there seems to be a shift in the value of critical mass flux as the supercharge is increased for injector number 1. Another interesting result that may be related is shown in Figure 20, which plots the critical $\Delta P$ versus supercharge for the same three injectors using nitrous oxide. For injectors number 2 and 5, there is nearly no difference in the critical $\Delta P$ value for most of the range of supercharge. Injector 1 has a similar value of critical $\Delta P$ to injectors 2 and 5 at low supercharge levels, but this value quickly drops, and ends up at a value approximately 100 psi lower at high supercharges. Both of these results may be due to Reynolds number effects or hydraulic flip, and will be investigated in future work.
Figure 19: Critical mass flux vs. supercharge for injectors number 1, 2 and 5 with nitrous oxide.

Figure 20: Critical $\Delta P$ vs. supercharge for injectors number 1, 2 and 5 with nitrous oxide.

COMPARISON OF RESULTS WITH NITROUS OXIDE AND CARBON DIOXIDE

Next we will take a look at some results that will help to determine whether or not carbon dioxide can be used as an effective analog to nitrous oxide. In contrast to the results presented above, these tests will be for a single injector geometry, and the fluid will be varied. Injector number 3 was used for this analysis. Figure 21 shows a compilation of the mass flow rate data used to compare the performance of nitrous oxide and carbon dioxide. Tests using carbon dioxide are plotted in blue while tests using nitrous oxide are plotted in red. This color scheme will be used for the remainder of this section. From examining this plot, it is clear that the overall character of the carbon dioxide performance mirrors that of the nitrous oxide quite well. Looking closely, it does seem that the critical $\Delta P$ and critical mass flow rates do match relatively closely between the two fluids, however because each test was performed at different supercharge levels, it is difficult to make any conclusions about the similarities using this figure alone.

For this reason, Figure 22 plots the critical mass flow rate versus supercharge for both carbon dioxide and nitrous oxide, and this type of comparison is made much more easily. It can
be seen that the critical mass flow rate of carbon dioxide does tend to be slightly higher than for nitrous oxide. However, the difference in critical mass flow rate between the two fluids is less than 10% for the whole range of supercharge values tested. Additionally, Figure 23 compares the discharge coefficient for the single phase region of these tests and for most supercharge levels the value is identical. At points where the discharge coefficient does vary between carbon dioxide and nitrous oxide, the values generally differ by less than 5%. It can be concluded from the results presented in Figure 22 and Figure 23 that carbon dioxide is indeed a relatively accurate analog, though small differences in performance do exist. Future testing may serve to provide correlations between nitrous oxide and carbon dioxide performance.

![Figure 21: Mass flow rate vs. ∆P and supercharge for injector number 3 using both nitrous oxide (red) and carbon dioxide (blue).](image1)

![Figure 22: Critical mass flow rate vs. supercharge for injector number 3 using nitrous oxide and carbon dioxide.](image2)
EFFECT OF INJECTOR HOLE INLET GEOMETRY

The last series of experimental comparisons presented in this work are performed using carbon dioxide, since we have determined in the previous section that it does indeed perform well as an analog to nitrous oxide. These tests examine the effect of varying the orifice inlet geometry on the mass flow rate performance. The injectors used for this testing all have a minimum cross-sectional diameter of 0.059", but one has a square edged inlet, one is rounded, and the last has a chamfer (injectors number 2, 3, and 4 respectively). As with the previous analyses, we will compare the injector performance in the single phase region and the two-phase region separately. Figure 24 shows the SPI discharge coefficient versus supercharge for the three different inlet geometries. It is clear from this plot that the square edged orifice exhibits a $C_d$ that is significantly lower than that of the rounded and chamfered injector, which is expected. Additionally, the $C_d$ value for the rounded injector appears to be slightly higher than that of the chamfered injector; however the difference is less than the uncertainty in the measurement.

Figure 25 compares the critical mass flow rate of the same three injectors, and results in similar findings. The critical mass flow rate in the square edged injector is approximately 20% lower than that of the rounded and chamfered injector, which display nearly identical performance. In addition to serving as a useful design tool, Figure 24 and Figure 25 demonstrate a significant result: while the mass flow rate of an injector hole can be increased dramatically by installing a simple chamfer, there is almost no additional improvement by going through the extra effort to round the edges instead.

Figure 26 plots the critical $\Delta P$ values versus supercharge for injector numbers 2, 3, and 4. This final plot of experimental data simply shows that the inlet geometry has very little effect on the onset of the critical flow regime.
Figure 24: $C_D$ in the single phase region vs. supercharge for injectors number 2, 3, and 4 using carbon dioxide.

Figure 25: Critical mass flow rate vs. supercharge for injectors number 2, 3, and 4 using carbon dioxide.

Figure 26: Critical $\Delta P$ vs. supercharge for injector numbers 2, 3, and 4 using carbon dioxide.
In order to validate the NHNE model as presented earlier in this work, the experimental and predicted critical mass flow rates are compared. Tests using each injector and both nitrous oxide and carbon dioxide have been included in these comparisons. Figure 27(a) shows the resulting comparison of the experimental critical mass flow rate to the NHNE, when a \( C_d \) value of 0.66 is used, as in the work of Dyer et al. It can be seen that assuming a constant value of discharge coefficient results in reasonable agreement for a number of conditions, but a significant number of tests that lie outside of the region bounded by ±10% error. However when the value of \( C_d \) is allowed to vary by injector design (0.59 to 0.81, depending on injector design), the NHNE does much better. Not only are almost all of predicted critical values within ±10% of the experimental values, but a majority lie within the ±5% bounds. While these improvements are encouraging, there is still plenty of work to be done to improve our capabilities in the modeling of self-pressurizing propellant injectors.

![Comparison of experimental mass flow data to that predicted by the NHNE model.](image)

Figure 27: Comparison of experimental mass flow data to that predicted by the NHNE model. (a) \( C_d = 0.66 \) (b) \( C_d \) determined from experimental data in the single phase flow region.

### SUMMARY AND CONCLUSIONS

This work has described a new experimental testing facility for the investigation of hybrid rocket injectors for use with self-pressurizing propellants, specifically nitrous oxide. This paper was split into three parts:

1) A background into the commonly used methods for predicting hybrid rocket injector mass flow rate. The lack of reliable discharge coefficient data or extensive validation for these models was used to provide a motivation for the experimental work.

2) A description of the test facility and the test method for making mass flow measurements was provided. Due to safety concerns, a discussion of the use of carbon dioxide as an analog to nitrous oxide was presented.

3) Initial experimental results were presented and compared to the aforementioned models to within 5-10%. The validity of using carbon dioxide as an analog to nitrous oxide was also confirmed.

4) The variations in performance for different injector designs were compared as outlined below:
   - \( C_d \) in the single phase region is inversely proportional to hole diameter, while the effect of hole diameter on critical mass flux varies with supercharge.
• Mass flow rate is enhanced by rounding or chamfering the inlet for both the single-phase and two-phase regions, however almost no improvement is seen by switching from chamfered to rounded.

• Inlet geometry has very little to no effect on the $\Delta P$ required for onset of critical flow

**FUTURE WORK**

The data from this work will be compiled into a database and a user interface will be developed to assist in hybrid rocket injector design. Injector mass flow rate performance characterization is ongoing and will investigate the effects of the following:

- Injector length
- Injector length to diameter ratio
- Angled holes
- Helium desorption
- Unconventional hole geometry

Another important characteristic of injectors for use in hybrid rocket systems is the degree to which the injector isolates or dampens the communication of pressure fluctuations upstream from the combustion chamber to the feed system. When combustion instabilities result in large amplitude pressure oscillations in the combustion chamber, it is undesirable for those pressure oscillations to be propagated upstream through the injector holes. This can result in feed system pressure oscillations and can be a safety concern with regards to the increased risk for nitrous oxide decomposition explosions in the feed line (if significant levels of vapor are produced upstream of the injector element). Additionally, interaction of pressure oscillations in the feed system and the combustion chamber can lead to undesirable low-frequency feed-coupled instabilities which are encountered in many large scale hybrid rocket systems. It is hypothesized that with sufficient cavitation and two-phase choking within in the injector elements, significant pressure isolation can be achieved by the injector itself. In order to assess the pressure isolation characteristics of the various injector geometries and operating conditions to be tested, a fast acting motor operated ball valve will be installed in the downstream pressurization line, and will be opened and closed periodically at rates of up to 50 Hz in order to establish the corresponding pressure oscillations in the chamber downstream of the injector. The pressure in the pre-injector volume will be monitored for each injector and the level of attenuation of the oscillating pressure signal will be compared at a wide variety of operating conditions.

**ACKNOWLEDGMENTS**

This work was supported by the National Defense Science and Engineering Graduate Fellowship Program (NDSEG) as well as NASA Ames Research Center, and the Aeronautics and Astronautics Department at Stanford University. Portions of this work were funded under NASA Space Technology Research Grant #NNX11AN14H. The authors would like to thank Jeremy Corpening of Teledyne Brown Engineering and Harry Ryan of NASA Stennis Space Center for their assistance, and David Murakami of NASA Ames Research Center for sacrificing his personal time to assist with some weekend testing.

**REFERENCES**


### APPENDIX A: REFERENCE TEST DATA

#### Table 3: Description of injector design geometry

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#### Table 4: Outline of figures included as reference data.

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<td>30</td>
<td>N(_2)O</td>
<td>(\dot{m})</td>
<td>2</td>
<td>10 – 690</td>
<td>90 – 422</td>
<td>505 – 510</td>
<td>618 – 991</td>
</tr>
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<td>(C_d)</td>
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<td>90 – 422</td>
<td>505 – 510</td>
<td>618 – 991</td>
</tr>
<tr>
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<td>(\dot{m})</td>
<td>2</td>
<td>0 – 605</td>
<td>104 – 301</td>
<td>502 – 515</td>
<td>686 – 955</td>
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<td>502 – 515</td>
<td>686 – 955</td>
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<tr>
<td>34</td>
<td>N(_2)O</td>
<td>(\dot{m})</td>
<td>3</td>
<td>0 – 675</td>
<td>41 – 371</td>
<td>505 – 510</td>
<td>568 – 937</td>
</tr>
<tr>
<td>35</td>
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<td>(C_d)</td>
<td>3</td>
<td>0 – 675</td>
<td>41 – 371</td>
<td>505 – 510</td>
<td>568 – 937</td>
</tr>
<tr>
<td>36</td>
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<td>3</td>
<td>0 – 700</td>
<td>59 – 317</td>
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<td>671 – 956</td>
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<tr>
<td>37</td>
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<td>(C_d)</td>
<td>3</td>
<td>0 – 700</td>
<td>59 – 317</td>
<td>506 – 510</td>
<td>671 – 956</td>
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<tr>
<td>38</td>
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<td>(\dot{m})</td>
<td>4</td>
<td>0 – 705</td>
<td>85 – 372</td>
<td>506 – 510</td>
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<tr>
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<td>(C_d)</td>
<td>4</td>
<td>0 – 705</td>
<td>85 – 372</td>
<td>506 – 510</td>
<td>687 – 1008</td>
</tr>
<tr>
<td>40</td>
<td>N(_2)O</td>
<td>(\dot{m})</td>
<td>5</td>
<td>0 – 560</td>
<td>78 – 420</td>
<td>507 – 510</td>
<td>623 – 988</td>
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<tr>
<td>41</td>
<td>N(_2)O</td>
<td>(C_d)</td>
<td>5</td>
<td>0 – 560</td>
<td>78 – 420</td>
<td>507 – 510</td>
<td>623 – 988</td>
</tr>
</tbody>
</table>
Figure 28: Mass flow rate vs. $\Delta P$ and supercharge for injector number 1 with nitrous oxide.

Figure 29: $C_d$ vs. $\Delta P$ and supercharge for injector number 1 with nitrous oxide.
Figure 30: Mass flow rate vs. $\Delta P$ and supercharge for injector number 1 with carbon dioxide.

Figure 31: $C_p$ vs. $\Delta P$ and supercharge for injector number 1 with carbon dioxide.
Figure 32: Mass flow rate vs. $\Delta P$ and supercharge for injector number 2 with nitrous oxide.

Figure 33: $C_d$ vs. $\Delta P$ and supercharge for injector number 2 with nitrous oxide.
Figure 34: Mass flow rate vs. $\Delta P$ and supercharge for injector number 2 with carbon dioxide.

Figure 35: $C_d$ vs. $\Delta P$ and supercharge for injector number 2 with carbon dioxide.
Figure 36: Mass flow rate vs. \( \Delta P \) and supercharge for injector number 3 with nitrous oxide.

Figure 37: \( C_d \) vs. \( \Delta P \) and supercharge for injector number 3 with nitrous oxide.
Figure 38: Mass flow rate vs. $\Delta P$ and supercharge for injector number 3 with carbon dioxide.

Figure 39: $C_J$ vs. $\Delta P$ and supercharge for injector number 3 with carbon dioxide.
Figure 40: Mass flow rate vs. \( \Delta P \) and supercharge for injector number 4 with carbon dioxide.

Figure 41: \( C_f \) vs. \( \Delta P \) and supercharge for injector number 4 with carbon dioxide.
Figure 42: Mass flow rate vs. $\Delta P$ and supercharge for injector number 5 with nitrous oxide.

Figure 43: $C_d$ vs. $\Delta P$ and supercharge for injector number 5 with nitrous oxide.