Initial Experimental Investigations of Self-Pressurizing Propellant Dynamics

Jonah E. Zimmerman* and Brian Cantwell†
Stanford University, Stanford, CA, 94305, USA

Gregory Zilliac‡
NASA Ames Research Center, Moffet Field, CA, 94035, USA

Hybrid rockets commonly use nitrous oxide as a self-pressurizing oxidizer, but no comprehensive models exist for predicting the performance of such a system. To aid in the development of such a model, an experimental apparatus has been constructed with a transparent cylinder that allows for optical measurements as well as pressure, temperature, and mass flowrate measurements. Carbon dioxide has been identified as an analog for nitrous oxide, facilitating laboratory-scale testing. Initial results of tests with this system show that there is significant boiling within the liquid and that the propellant is not maintained in thermodynamic equilibrium. Furthermore, both supercharge configuration and mass flowrate strongly affect the dynamics of self-pressurizing propellants.

Nomenclature

- $\mu$: dynamic viscosity
- $\sigma$: surface tension
- $k$: thermal conductivity
- $Z$: liquid or vapor compressibility
- $\rho$: density
- $h_{lv}$: latent heat of vaporization
- $C_P$: isobaric specific heat capacity
- $C$: flow coefficient
- $C_d$: discharge coefficient
- $A$: area
- $P$: pressure
- $R$: specific gas constant
- $T$: temperature
- $\gamma$: ratio of specific heats
- $Y'$: general compressibility correction
- $Y$: ideal gas compressibility correction

Subscript

- $r$: reduced
- $c$: critical point value
- $liq$: liquid
- $vap$: vapor
- $t$: throat
- $T$: plenum
- $LRO$: point at which the liquid runs out
- $norm$: normalized

I. Introduction

Nitrous oxide is an attractive oxidizer for hybrid rockets due to its ease of handling, low toxicity, and high vapor pressure. It can be configured as a self-pressurizing propellant by utilizing its vapor pressure to drive the propellant into the combustion chamber. This arrangement eliminates the need for a pressurization system or turbopump, but generates other complications. The fluid mechanics and thermodynamics of this emptying tank are complex, and without an accurate model for this system the prediction of the oxidizer flowrate is impossible. As an example, figure 1 shows pressure histories from a motor using self-pressurized N$_2$O.1 The oxidizer tank pressure is seen to initially decrease steeply, but the slope increases after approximately 0.75 s.

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*PhD Candidate, Department of Aeronautics & Astronautics, AIAA Member (jonahz@stanford.edu).
†Professor, Department of Aeronautics & Astronautics, AIAA Fellow.
‡Research Scientist, NASA Ames Research Center, AIAA Member.
Casalino & Pastrone\textsuperscript{2} demonstrated the implications of inaccurate models by optimizing the design of N$_2$O - HTPB sounding rockets using two different models of propellant tank dynamics for nitrous oxide. They found that the two optimized vehicles had differing thrust, O/F ratio, oxidizer temperature, and acceleration throughout the burn. The thrust histories of their optimized motors are shown in figure 2, where the two models are labeled “homogeneous” and “two-phase.”

Generally speaking, many levels of approximation may be used to model the self-pressurizing N$_2$O system. By assuming a saturated state where the liquid and vapor remain in phase equilibrium, the entire thermodynamic state of the tank may be represented by a single variable, and the inclusion of an extensive variable allows the complete description of the propellant within the tank. Furthermore, by either invoking an adiabatic assumption or using correlations for heat transfer to and from the walls of a cylinder, the model becomes a closed system of ordinary differential equations and can be integrated to yield the time histories of pressure, temperature, and mass flowrate.

Whitmore & Chandler\textsuperscript{3} developed such a model for a self-pressurizing propellant system using entropy and vapor quality ($m_{vap}/(m_{vap} + m_{liq})$) as the variables. Using this model they accurately predicted experimental temperature, pressure, and mass flow data results for an N$_2$O tank being vented to ambient pressure (figure 3). The experimental data were obtained by venting N$_2$O vapor for a period of time, dropping the temperature and pressure significantly, prior to draining liquid from the tank.

Non-saturated models have also been developed. Here, the liquid and vapor are both assumed to be uniform but at different temperatures, and heat and mass transfer between the two phases must be directly calculated in order to close the system. Additionally, while the mathematical system is still constructed with ordinary differential equations, the increasing complexity requires a more advanced numerical solution.

Ziliac & Karabeyoglu\textsuperscript{4} developed such a model for self-pressurizing propellants based on the earlier work of Morey &
Traxler for the pressurization of classical liquid fuels and oxidizers. Two noteworthy assumptions are used this work. The first is that the only mass transfer between the liquid and vapor occurs through vaporization from the surface of the liquid and condensation to the surface of the liquid. Second, the heat transfer between the liquid and vapor follows empirical models for natural convection from flat plates. This model was compared to experimental data obtained from a N₂O tank venting to ambient pressure and was found to accurately predict the pressure and mass histories (figure 4).

More advanced models that do not assume that the liquid or vapor states are uniform have yet to be developed for this system. While the models of Zilliac & Karabeyoglu and Whitmore & Chandler are able to predict their own experiments, they are founded on fundamentally different assumptions and it is unclear which model is applicable to a given propulsion system. Therefore, a broader understanding of the system and the regimes in which these and other assumptions can be made is desired. To this end, an experimental apparatus was designed and constructed that facilitates detailed measurements of self-pressurizing propellant tanks. This system and initial results from it are described in the following sections of this paper.

II. Carbon Dioxide as an Analog

Working with nitrous oxide in a university laboratory setting is problematic as it is both a strong oxidizer and can decompose exothermically. These decomposition events have even caused fatal explosions. While these hazards are manageable through materials selection and procedural precautions, a substitute would be desirable to mitigate safety risks and reduce costs.

Carbon dioxide is similar to nitrous oxide in many respects, which suggests that for this research it could be used as an analog for laboratory-scale testing. A comparison of various thermodynamic properties of the two chemicals is given in table 1. With the exception of the triple point and the acentric factor, all the properties are within 5.5% of each other. The acentric factor is a convenient way of characterizing how different a fluid behaves compared to simple monatomic fluids such as argon or krypton. It is closely related to the nonsphericity of a molecule’s potential field. The differing acentric factors suggest that some deviations in thermodynamic properties could be expected, as evidenced by the triple point pressures.

To explore the differences between the two fluids, figures 5 and 6 show the relative difference in 11 thermodynamic and transport properties as a function of reduced temperature in the range of interest. Here, relative difference is defined as \((X_{\text{CO}_2} - X_{\text{N}_2\text{O}})/X_{\text{N}_2\text{O}}\). Although the critical temperatures of the two fluids only differ by 5.4 K, all thermodynamic properties vary wildly near the critical point and this small temperature difference can create large differences in the various fluid properties. Therefore reduced temperature, \(T_r = T/T_c\), is used as the independent variable. Note that the range of \(T_r\), 0.75 to 1, encompasses the full temperature range encountered during testing (\(\sim 230\)K to \(\sim 310\)K).

Of the properties plotted, the thermodynamic properties \((P, \rho, Z, h_{lv}, C_P)\) show little difference and are usually within 10% for the two fluids. The transport properties \((\mu, k)\) show larger differences, with the thermal conductivity differing by up to 100% between carbon dioxide and nitrous oxide. Despite this, the viscosity is within 50% for the entire range and the thermal conductivity is within 50% for most of the range of interest.

From these analysis results we can conclude that most properties of the two fluids are very similar and therefore carbon dioxide should function as an accurate analog for nitrous oxide. Some transport properties show moderate differences, and if the dynamics of self-pressurizing propellants are strongly dependent upon these particular properties there may be some differences in test results with CO₂ and N₂O. Testing with both fluids will be completed in the future in order to directly evaluate CO₂’s accuracy as an analog.
### Table 1. Comparison of N$_2$O and CO$_2$ thermodynamic properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>N$_2$O</th>
<th>CO$_2$</th>
<th>% Difference</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular Weight [g mole$^{-1}$]</td>
<td>44.0128</td>
<td>44.0098</td>
<td>-0.0068</td>
<td>8,9,10</td>
</tr>
<tr>
<td>Critical Temperature [K]</td>
<td>309.52</td>
<td>304.1282</td>
<td>-1.7</td>
<td>8,11</td>
</tr>
<tr>
<td>Critical Density [kg m$^{-3}$]</td>
<td>452.0</td>
<td>467.6</td>
<td>3.4</td>
<td>8,11</td>
</tr>
<tr>
<td>Critical Pressure [MPa (psi)]</td>
<td>7.245 (1051)</td>
<td>7.3773 (1070.0)</td>
<td>1.8</td>
<td>8,11</td>
</tr>
<tr>
<td>Triple Point Temperature [K]</td>
<td>182.33</td>
<td>216.592</td>
<td>19</td>
<td>12,11,10</td>
</tr>
<tr>
<td>Triple Point Pressure [MPa (psi)]</td>
<td>0.08791 (12.75)</td>
<td>0.51795 (75.122)</td>
<td>490</td>
<td>13,11,10</td>
</tr>
<tr>
<td>Boiling Point [K]</td>
<td>184.68</td>
<td>194.75</td>
<td>5.5</td>
<td>8,10</td>
</tr>
<tr>
<td>Acentric Factor</td>
<td>0.1613</td>
<td>0.22394</td>
<td>39</td>
<td>8,10</td>
</tr>
</tbody>
</table>

![Figure 5. Comparison of saturated nitrous oxide and carbon dioxide properties. Data produced using References 11 and 8.](image)

### III. Experimental Setup

In order to investigate the dynamics of self-pressurizing propellant tanks in detail, an experimental system was developed using a transparent polycarbonate tube as a mock propellant tank that can be filled and then vented, simulating the conditions of a rocket motor firing. Because the tank is transparent, the presence of boiling, convection cells, and other processes can easily be determined using video imagery. A picture of the tank is shown in figure 7 and a piping and instrumentation diagram is given in Appendix A, along with details of the data acquisition system.

The tank has a 25.4mm (1 in) inner diameter with 6.35 mm (0.25 in) walls, is 356 mm (14 in) long, and has a volume of 180,200 mm$^3$ (11.0 in$^3$). At room temperature (293 K) the tank holds 0.14 kg (0.31 lbm) of liquid carbon dioxide. During testing at high flowrates, the tank can be emptied in approximately 2 s. All pipes in the system are constructed from 6.35mm (0.25 in) stainless steel tubing.

The tank is filled from a supply K-cylinder using a pneumatically-powered pump designed for use with
nitrous oxide. Gaseous CO$_2$ can be vented through the top of the tank. During a test, the CO$_2$ flows down out of the tank and past a K-type thermocouple that monitors the effluent temperature. It then flows through a venturi, where a pressure transducer measures the static pressure and a differential pressure transducer is used to compute the flowrate (see Appendix B for detail on using venturi flowmeters with compressible liquids). The main valve controlling the flowrate is a pneumatically-actuated ball valve located just downstream of the venturi.

From the exit of this main valve, the CO$_2$ flows into an injector testing setup that has been incorporated into this system. Results from injector tests are discussed elsewhere,$^{14}$ but for the tank tests in this work the injector testing system was used to control the mass flowrate and maintain a low pressure downstream of the injector. When the flow enters this setup, it first passes through a diffuser into the pre-injector volume, a section of polycarbonate tubing that has the same diameter as the tank. Then it passes through an injector and enters the chamber, another piece of polycarbonate tubing that is longer but of the same diameter. Another pneumatically-actuated ball valve controls the exit from the chamber, where the CO$_2$ flows through a metering needle valve before venting to ambient conditions.

A digital camera is used to record video of the tank, and a laser sheet is used to help visualize the flowfield by illuminating bubbles and condensing vapor. The laser is a standard green (532 nm) laser pointer whose beam passes through a glass rod in order to create a laser sheet. This laser sheet is directed through the centerline of the tank and perpendicular to the camera’s viewing direction.

Frequently with nitrous oxide-based hybrids the tank pressure is increased via the addition of an inert gas, typically helium. This is referred to as supercharging and may serve several purposes. With a propellant that is near the saturation state small decreases in static pressure such as those caused by losses in the feed system can bring the static pressure below the saturation pressure and induce unwanted cavitation. By adding helium to the tank and maintaining the pressure above the saturation pressure this cavitation can be eliminated. Secondly, N$_2$O vapor can decompose exothermically, possibly resulting in an explosion. The addition of an inert gas to the vapor can greatly reduce the risk of such decomposition events.$^6$ Therefore, helium is used to increase the tank pressure in some tests and is supplied from a standard K-cylinder through the top of the tank.
Discussed throughout this work are the video recordings and the measured static pressure and temperature. As mentioned earlier, the pressure and temperature are both measured in the tubing just downstream of the tank. To a good approximation, the pressure measured here can be taken as the pressure within the tank because the fluid velocities are small (< 20 m/s). Given the proximity of the temperature probe to the base of the tank, the temperature measured here is taken as the temperature of the liquid at the base of the tank before a test has begun when there is no flow. The accuracy of this approximation was confirmed by the observation that when the tank was filled with liquid CO\textsubscript{2} the saturation pressure calculated based on this temperature quickly approached the measured pressure. Once the main flow valve is opened and CO\textsubscript{2} is draining from the tank, this temperature is only considered as the effluent temperature.

While the venturi allows for a measurement of the mass flowrate, two-phase flow was common in many tests and disrupted this calculation. Therefore, to facilitate direct comparison of all configurations the venturi measurement was not used in this work. In order to compute an averaged mass flowrate, the mass of liquid initially present in the tank is divided by the time required to transition from liquid to gas flow out of the tank:

\[
\dot{m} = \frac{m_{\text{liq,initial}}}{t_{\text{LRO}}} \quad (1)
\]

Figure 7. Picture of the tank used in this work, constructed from a polycarbonate tube with aluminum end caps.

Where LRO refers to the point at which the liquid is completely expelled and gas begins to flow out of the tank. Parameters that can be varied with this setup include:

- **Initial Fill Level:** The amount of CO\textsubscript{2} that is pumped into the tank controls the initial fill level.
- **Mass Flowrate:** Installing different sized injectors or controlling the metering valve alters the mass flowrate.
- **Initial Temperature:** Venting CO\textsubscript{2} vapor from the top of the tank causes the liquid CO\textsubscript{2} to boil and vaporize, absorbing heat via the phase change process and reducing the liquid temperature.
- **Initial Temperature Distribution:** A pump is used to move liquid CO\textsubscript{2} into the tank and therefore the gas temperature rises due to compression work. This causes the liquid CO\textsubscript{2} to be stratified, with warmer CO\textsubscript{2} at the top and cooler CO\textsubscript{2} at the bottom. The thick polycarbonate walls limit heat transfer into the tank, and hence inhibit the establishment of convection cells. However, this stratification can be minimized by either venting gaseous CO\textsubscript{2} or using a cooling jacket.

- **Supercharge Level:** Helium may be added to the tank to increase the total pressure by a set amount.
- **Supercharge Configuration:** If the helium valve remains open during a test the tank pressure remains fairly constant, but a second configuration involves adding helium prior to the test and then closing this valve so that there is only an initial supercharge.

Although varying these parameters yields a wide range of testing conditions, there are additional parameters that could influence results but can not be easily varied with this setup. Most importantly is the tank size and geometry. The overall size of the tank is likely to be important because many of the non-dimensional...
numbers that characterize fluid mechanics, heat transfer, and mass transfer are dependent on length scales. Secondly, if the heat and mass transfer between the vapor and liquid only occur at the interface as hypothesized by Ref. 4, then the ratio of the area of that interface to the volumes of liquid and vapor would control the impact of heat and mass transfer between the phases. Therefore, changing the length to diameter ratio of the tank may also effect the dynamics. Lastly, heat transfer to and from the walls may be important to the overall dynamics so changing either the tank wall thickness or its thermal properties may have some effect.

IV. Results

The parameters listed in the previous section present a 6-dimensional space, and thus a large number of tests will be required to fully characterize the self-pressurizing propellant tank system. For this work, only the mass flowrate and supercharge configuration were varied in order to obtain a tractable test matrix. Therefore all tests were run with approximately the same initial conditions.

The liquid level (90%), temperature (287 K), CO\textsubscript{2} pressure (5.2 MPa (750 psi)) and helium pressure (6.9 MPa (1000 psi)) were chosen to replicate typical conditions within a hybrid rocket. Additionally mass flowrate was varied from 0.004 kg/s to 0.07 kg/s in order to obtain test durations that are similar to burn times in hybrid sounding rockets, 2 s to 30 s.

A. Tests at Varying Mass Flowrate

Here tests are presented for each of the three supercharge configurations as mass flowrate is varied.

1. Non-Supercharged Tests

In these tests no helium was added to alter the pressure of the tank. The test procedure begins with pumping liquid CO\textsubscript{2} into the tank until it reaches a pre-determined height. Once the temperature field becomes uniform, the main valve is opened and liquid CO\textsubscript{2} flows out the bottom of the tank. In figure 8 typical pressure and temperature histories are shown and in figure 9 stills are shown from the video of this test.

![Figure 8. Typical pressure and temperature histories for a non-supercharged test.](image)

The pressure history begins with a steep decrease, followed by a slight increase to a peak at \(\sim\) 2 s. From there it decreases again, with a decrease in slope at \(\sim\) 10 s, followed by a slow increase in slope until the end of the test. This non-monotonic decrease in pressure is always visible with tests from this system and has been reported elsewhere\(^4\) but is not always present in data from N\textsubscript{2}O rocket tests (see figure 1). The temperature history shows somewhat similar behavior, although there is a minimum during the later phase of the test at \(\sim\) 18 s, followed by an increase in temperature.

The video stills shown in figure 9 allow some identification of the dynamic processes underway in the tank. In the first image, the main valve is opened and the vapor above the liquid immediately condenses,
making it visible as a bright green band at the top. In the next image, bubbles are seen rising from the bottom of the tank towards the free surface. In the third image these bubbles reach the surface and from this point onwards the liquid appears to be densely and uniformly populated with bubbles. The liquid level then drops until the liquid has been expended.

When the plots and images are compared, several features become apparent. First, the change in slope of the pressure curve at $\sim 10$ s corresponds to the point when the liquid has been completely expelled from the tank. Secondly, the point at $\sim 2$ s where the pressure reaches a maximum is the point when the bubbles that rise from the bottom of the tank reach the free surface. Third, the almost-linear decrease of pressure and temperature with time from $\sim 2$ s to $\sim 10$ s corresponds to the flow of the bubbly CO$_2$ out of the tank. While these features are described here for a specific test, they have been observed in all tests discussed in this section.

Figures 10 and 11 show several pressure and temperature histories normalized and overlaid. Data are only shown for the portion of the test in which liquid is flowing out of the tank. The data are normalized as

$$P_{\text{norm}} = \frac{P - P_{LRO}}{P_{\text{initial}} - P_{LRO}}, \quad T_{\text{norm}} = \frac{T - T_{LRO}}{T_{\text{initial}} - T_{LRO}}, \quad t_{\text{norm}} = \frac{t}{t_{LRO}}, \quad \dot{m}_{\text{norm}} = \frac{\dot{m}}{\dot{m}_{\text{min}}}$$

The pressure, temperature, and time are normalized in this way in order to evaluate how the fundamental character of the pressure and temperature histories is altered by varying the mass flowrate. Mass flowrate is calculated as described in Eq. (1), and is normalized by the smallest mass flowrate recorded in this work, $\dot{m}_{\text{min}} = 4.4$ g/s.

In figure 10, it is apparent that after the initial dip all the pressure curves are approximately linear and have similar slopes. The magnitude and duration of the dip also increases with increasing mass flowrate. In figure 11, the point at which the temperature curve becomes linear is delayed as mass flowrate increases and corresponds to the peak in the pressure history.

In figure 12, the temperature and pressure recorded at $t = t_{LRO}$ are shown as a function of the mass flowrate. Here the values are normalized as

$$P_{\text{norm},LRO} = \frac{P_{LRO}}{P_{\text{initial}}}, \quad T_{\text{norm},LRO} = \frac{T_{LRO}}{T_{\text{initial}}}$$
The normalization scheme is altered because all values at $t_tLRO$ would be zero using Eq. (2). Both the temperature and pressure at $t_tLRO$ decrease as mass flowrate is increased, with the pressure in particular decreasing significantly.
2. Initially Supercharged Tests

In this section results are presented for tests in which helium has been added to the tank, but only prior to the test and not while liquid CO$_2$ is drained from the tank. The test procedure begins with again pumping liquid CO$_2$ into the tank until it reaches a pre-determined height. Then, the pressure is increased by adding helium until the pressure in the tank reaches the level supplied by the regulator. For these tests this level is approximately 6.9 MPa (1000 psi), raising the pressure by $\sim$1.7 MPa ($\sim$250 psi). The helium valve is then closed and the main valve is opened until the tank is drained.

![Figure 13. Typical pressure and temperature histories for an initially supercharged test.](image)

In figure 13 typical pressure and temperature histories are shown and in figure 14 stills are shown from the video of this test. The pressure plot is somewhat similar to that shown in figure 8 for non-supercharged tests, but the initial pressure decrease is much greater in magnitude ($\sim$2 MPa (300 psi) instead of $\sim$0.6 MPa (100 psi)) and there is no subsequent pressure rise, only a very slow decrease. This is followed by an almost linear decrease similar to that seen in the non-supercharged test. The temperature traces in figures 13 and 8 are quite similar.

The video stills shown in figure 14 show differences compared to the non-supercharged test, especially in the first 3 s. In the first image, the main valve is opened and a small amount of vapor immediately above the liquid surface condenses, making it visible. In the next two stills, this condensed vapor region grows and some bubbles form in the very top portion of the liquid. Then, both of these regions appear to dissipate as the liquid level is almost stationary and bubbles rise from the bottom of the tank. The bubbles then reach the free surface and the rest of test continues similarly to the non-supercharged test (figure 9).
Figure 14. Stills from video of an initially supercharged test. The white numbers indicate the time in seconds from the start of the test.

The main features of the pressure plots correspond to points in the video that are similar to those in the non-supercharged test. First, the steep decrease in slope at $\sim 11$ s corresponds to the liquid run out point. The flat portion of the curve, from $\sim 1$ s to $\sim 3$ s corresponds to the time in which the liquid surface height is almost constant, and the end of this portion of the curve corresponds to the point when the bubbles rising from the bottom of the tank reach the surface. This is similar to the non-supercharged tests, where the peak of the pressure curve aligned with bubbles reaching the free surface. While these features are described here for a specific test, they have been observed in all tests discussed in this section.

Figures 15 and 16 show several pressure and temperature histories that are normalized and overlaid. Again, data are only shown for the portion of the test in which liquid is flowing out of the tank. The data are normalized as described in Eq. (3). With the exception of one test (the line marked with $\triangledown$), all the data follow very similar trends. Like the non-supercharged tests, the magnitude of the initial pressure drop increases with increasing mass flowrate, although there appears to be more variation in the linear portion of the curve for the initially supercharged tests.

In figure 16, there is more variation in the normalized temperature histories than for the non-supercharged tests. In one test, the temperature increases after the start of the test before decreasing in a fashion similar to the other tests. In both figures 15 and 16, the features of the highest mass flowrate test ($\triangledown$) are different from the other tests. In figure 15, the initial pressure drop is much smoother, and in figure 16 there is no dip whatsoever. This could signify that at this elevated mass flowrate some regime boundary has been crossed, or simply may be the result of variation in an unknown parameter.

In figure 17, the temperature and pressure recorded at $t = t_{LRO}$ are shown as a function of the mass flowrate. The values are normalized as shown in Eq. (3). The trends here are very similar to those in the non-supercharged tests; both the temperature and pressure at liquid run out decrease with increasing mass flowrate.
3. Continuously Supercharged Tests

In this section, results are presented for tests where helium is continuously added to the tank in an attempt to maintain the pressure at approximately 6.9 MPa (1000 psi) for the duration of a test. The helium supply
system wasn’t capable of a flowrate required to accomplish this and therefore there were some pressure fluctuations. This situation is not uncommon to hybrid rockets. In figure 18, typical pressure and temperature histories are shown, and in figure 19 stills are shown from the video of this test.

The pressure and temperature histories for the continuously supercharged tests differ considerably from the non-supercharged and initially supercharged tests. The pressure decreases slightly by \( \sim 0.6 \text{ MPa (90 psi)} \) in the first 0.5 s, and then stabilizes at 6.9 MPa (1000 psi) until 4.5 s, at which point there is a significant decrease in pressure. The temperature plot increases gradually from the initial point to 4.5 s, where it increases by 1 K for 1 s and then decreases precipitously.

The video stills shown in figure 19 show a complete lack of bubbles or condensing vapor, in stark contrast to those in figures 9 and 14. Therefore the laser sheet was not utilized. The liquid remains stable and without nucleation for the duration of the test. This behavior was observed for all continuously supercharged tests.

When the video and pressure and temperature plots are compared, the only noticeable feature is that the steep drop in pressure and temperature at 4.5 s corresponds to the liquid being completely expelled from the tank. A possible cause for the temperature increase throughout the test is the result of liquid that is slightly stratified.

Initially, cold CO\(_2\) settles at the bottom of the tank, while warmer CO\(_2\) is at the top of the tank. This stratification is minimized by cooling the top of the tank and enabling convection cells, but some stratification likely remains. Therefore, if no other process is acting to change the liquid temperature, as the liquid is drained from the tank the thermocouple which measures the effluent will register a slight increase...
in temperature with time.

Additionally the sudden increase in temperature around 4.5 s when the liquid is nearly expended may be the result of initial warming of the liquid carbon dioxide. Because this liquid is in contact with the ullage gas, as helium enters prior to the test the gaseous CO$_2$ will be compressed and warm. Some of this heat may be transferred to the upper liquid, which then passes the thermocouple near the end of the test.

Figures 20 and 21 show plots of pressures and temperatures from several tests that have been normalized per Eq. (2). While it appears that there are large variations in both pressure and temperature, this is an artifact of the normalization process and the small values of $(P_{\text{initial}} - P_{LRO})$ and $(T_{\text{initial}} - T_{LRO})$. The average pressure during the test decreases with increasing mass flowrate, but this may be a result of the performance of the helium supply system. With the exception of the lowest flowrate test (marked by ◦’s), the normalized temperature appears to increase with increasing mass flowrate.

Figure 20. Overlaid normalized pressure histories for several continuously supercharged tests.

Figure 22 shows how the pressure and temperature, normalized per Eq. (3), vary with mass flowrate. There appears to be no consistent trend to the data.
B. Tests at Varying Supercharge Configuration

Here results are presented for the various supercharge configurations but with a fixed mass flowrate. Figures 23 and 24 show the pressure and temperature histories for three tests with different supercharge configurations that have an identical average mass flowrate. Pressure and temperature stay fairly constant for the continuously supercharged test, while both the non-supercharged and initially supercharged tests show pressures and temperatures that decrease significantly with time. With the exception of the first second of the test, the initially supercharged test pressure and temperature matches the non-supercharged test with an offset of \( \sim 0.3 \) MPa (40 psi) and 3 K to 5 K, respectively.

In figures 25 and 26 the pressure and temperature histories are again plotted, but normalized as per Eq. (2). The character of the normalized pressure curves are quite different, with the continuously supercharged test showing the most variation and the non-supercharged test showing the least.
Figure 23. Overlaid pressure histories from tests with each supercharge configuration but with the same averaged mass flowrate.

Figure 24. Overlaid temperature histories from tests with each supercharge configuration but with the same averaged mass flowrate.

It must be noted that although the continuously supercharged curves in both figures 25 and 26 appear to have a higher degree of noise, this is a result of the normalization process; the values of \( P_{\text{initial}} - P_{\text{LRO}} \) and \( T_{\text{initial}} - T_{\text{LRO}} \) are small for this configuration.

Table 2 compares the total observed variation in normalized pressure and temperature at \( t = t_{\text{LRO}} \) for the three configurations. The very low variation between the continuously supercharged tests indicates a very weak dependence of the dynamics on mass flowrate. Conversely, the other two configurations show much larger dependencies, although the non-supercharged and initially supercharged results are quite similar.

Table 2. Comparison of normalized pressure and temperature variation at liquid run out for various supercharge configurations.

<table>
<thead>
<tr>
<th></th>
<th>Non-Supercharged</th>
<th>Initially Supercharged</th>
<th>Continuously Supercharged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>0.18</td>
<td>0.19</td>
<td>0.026</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.022</td>
<td>0.020</td>
<td>0.0069</td>
</tr>
</tbody>
</table>
C. Stratification Effects

For the tests presented in the previous sections of this paper, variations in the temperature field were kept minimal by cooling the top of the tank and venting gaseous CO$_2$ periodically. However, initially it was not
known that this would be required to establish a uniform temperature field. Therefore, some data were recorded for tests in which the liquid CO$_2$ was significantly stratified.

Due to buoyancy forces the warmest liquid CO$_2$ is at the free surface, where it is in contact with the ullage. If this upper liquid is in equilibrium with the ullage vapor then the pressure in the tank is the saturation pressure based on the temperature in this region. Conversely the CO$_2$ at the bottom of the tank is the coldest and has a lower saturation pressure.

By measuring the temperature at the base of the tank a saturation pressure can be computed. If this saturation pressure at the base of the tank is lower than the measured pressure, then the CO$_2$ at the top of the tank must be warmer and stratification is present. For the work previously presented in this paper tests were begun once this temperature difference was less than 2 K, determined by a pressure difference of less than 0.17 MPa (25 psi).

In figure 27, the pressure history from a stratified non-supercharged test is compared to those of a uniform non-supercharged and a uniform initially supercharged test. It is apparent that the stratified test behaves very similarly to an initially supercharged test. In effect then, the liquid CO$_2$ within the tank has been supercharged with higher temperature carbon dioxide.

![Figure 27](image)

**Figure 27.** Overlaid pressure histories stratified and uniform initial temperature distributions.

V. Conclusions

An experimental system has been developed for investigating self-pressurizing propellant tank dynamics, and a detailed analysis has shown that CO$_2$ is likely to be an accurate analog for nitrous oxide. Initial tests with this system evaluated the effects of supercharge configuration and mass flowrate on the dynamics of the system. With these data, some specific conclusions can be drawn:

1. The supercharge configuration has a significant impact on the behavior of the system, as demonstrated in figures 23 - 26. Non-supercharged and initially supercharged tests have similar pressure and temperature histories after the initial steep decrease in pressure in the first second of the test. Continuously supercharged tests have pressures and temperatures that remain fairly constant. The normalized pressures and temperatures for the three configurations all have fundamentally different characters, with the non-supercharged tests showing the least variation and the continuously supercharged tests showing the most.
2. Mass flowrate affects pressure and temperature histories for non-supercharged and initially supercharged tests, but has a minimal effect on continuously supercharged tests as shown in table 2. A significant decrease in pressure and temperature at \( t = t_{LRO} \) was measured for the non-supercharged and initially supercharged tests, visible in figures 12 and 17. For the continuously supercharged tests there was no apparent dependence of either normalized pressure or temperature at liquid run out on mass flowrate.

3. Tanks that are stratified may behave as if they are initially supercharged as shown in figure 27. If the purpose of the supercharge is only to increase the static pressure above the saturation pressure in order to reduce cavitation, then warming the top of the tank or cooling the bottom may achieve the same results as helium addition. However if the purpose of initially supercharging is to mitigate the risk of decomposition events, then clearly stratification will not have the desired effect.

Additionally, it is possible to make some comments on the applicability of the models by Whitmore & Chandler and Zilliac & Karabeyoglu mentioned earlier. It appears that the liquid and vapor CO\(_2\) do not remain in phase equilibrium, and hence cannot be described by a saturation model such as that of Whitmore & Chandler. For a tank that is maintained at the saturation state where the propellant remains in thermodynamic equilibrium, there is no direct time dependence for any of the processes within the tank. Heat and mass are transferred between the phases, but the equilibrium assumption implies that these occur infinitely fast compared to the rate at which propellant is removed from the tank.

Therefore, the state within the tank will only depend on the amount of propellant that has been removed, and not the rate at which it was removed. In other words, when the liquid has been completely expelled there is only one possible value for pressure and temperature regardless of how fast the liquid was drained. However, the data in figure 12 clearly show a dependence of the pressure and temperature at \( t_{LRO} \) on the mass flowrate. Thus, the system used in these experiments is not maintained at the equilibrium saturation state. A similar argument can be made for figure 17, although the initially supercharged tests are complicated by the presence of a second component. Nothing can be conclusively stated about the continuously supercharged tests because helium is being added throughout the test at an unmeasured rate.

One caveat to this argument is that if there is significant heat transfer into the tank from the surroundings then a flowrate dependence will appear. This is because the thermodynamic state within the tank will now depend on the amount of heat transferred into the tank which in turn depends on how long the draining tank is exposed to ambient air. However, for the system used in this work an adiabatic condition is more likely due to the high thermal resistance of the polycarbonate walls and the short test times.

Based on the initial results from this experiment, it is apparent that there is significant boiling within the liquid for the non-supercharged and initially supercharged tests (see figures 9 and 14). This directly contradicts one of the assumptions in Zilliac & Karabeyoglu’s model, namely that the heat and mass transfer between the liquid and vapor only occur via vaporization and condensation at the liquid surface.

Despite the fact that the evidence here suggests that both these models may be inaccurate, the researchers in both cases were able to accurately predict their own experimental results. This in turn suggests that there may be a fundamental difference between the experiments presented in this work and those in References 3 and 4. One likely cause is the size scale - the tank used in this work is at least an order of magnitude smaller than either of those of the earlier works. Secondly, the differences between CO\(_2\) and N\(_2\)O may be significant enough to fundamentally change the dominant dynamic processes within the tank.

Therefore, future work on this topic will endeavor to include both tests at larger scales and comparison tests between N\(_2\)O and CO\(_2\) in addition to tests within the parameter space of this experimental system. These tests will help achieve the goal of identifying the regime boundaries and aid the development of a comprehensive self-pressurizing propellant model.

**Acknowledgments**

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Appendix A: Details of Experimental Setup

In table 3, details are given for the various components of the data acquisition system and in figure 28 a piping & instrumentation diagram is shown.

Table 3. Measurement system details.

<table>
<thead>
<tr>
<th>Name</th>
<th>Manufacturer</th>
<th>Model</th>
<th>Specifications</th>
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<tr>
<td>Helium Pressure Transducer</td>
<td>Senstronics</td>
<td>OL01084-002</td>
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<tr>
<td>All Other Static Pressure Transducers</td>
<td>Measurement Specialties Inc.</td>
<td>MS156-000004-01KPG</td>
<td>0-1000 psig, 4-20 mA</td>
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<tr>
<td>Venturi Thermocouple</td>
<td>Omega</td>
<td>KMQSS-062G-3</td>
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<td>Thermocouple Amplifier</td>
<td>Analog Devices</td>
<td>AD595CQ</td>
<td>~10 mV/°C</td>
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<tr>
<td>Differential Pressure Transducer</td>
<td>Stellar Technology Inc.</td>
<td>DT140-50UD-119</td>
<td>0-50 psid, 3mV/V</td>
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<tr>
<td>Differential Pressure Transducer Amplifier</td>
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<td>AD624ADZ</td>
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<tr>
<td>Digital Camera</td>
<td>Casio</td>
<td>EX-F1</td>
<td>60 fps, 1080p</td>
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<td>Data Acquisition Board</td>
<td>National Instruments</td>
<td>USB-6210</td>
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</tr>
</tbody>
</table>

Appendix B: Venturi Flowmeters with Compressible Liquids

A common arrangement for measuring propellant flowrates with rockets involves the use of a venturi flowmeter. With incompressible liquids the flowrate is calculated as

\[
\dot{m} = CA_t \sqrt{2 \rho \Delta P} \tag{4}
\]

Where \( \Delta P \) is the pressure difference between the entrance and throat. Correcting for compressibility effects with liquids is often completed through experimental testing. In situations where experimental data for the compressible liquid are not available or only available at low accuracy, a theoretical treatment would be desirable.

In Ref. 16, equations for the flowrate of non-ideal gases through a venturi are developed. For isentropic flows,

\[
\dot{m} = \frac{C_d A_t P_T}{\sqrt{Z_T R T}} \left[ \frac{2n}{n-1} \left( 1 - \frac{\Delta P}{P_T} \right) ^{\frac{n}{2}} \left[ 1 - \left( 1 - \frac{\Delta P}{P_T} \right) ^{\frac{n-1}{n}} \right] \right] \tag{5}
\]

where

\[
n = \gamma \left[ \frac{Z + T \left( \frac{\partial Z}{\partial T} \right)_P}{Z + T \left( \frac{\partial Z}{\partial T} \right)_P} \right] \tag{6}
\]

These equations are in terms of plenum values (subscript \( T \)), which can be taken as the venturi entrance values if the ratio of throat area to entrance area is very small. Eq.(5) can be applied directly to liquid \( \text{N}_2\text{O} \) or \( \text{CO}_2 \) because both substances can be described by the real gas equation of state

\[
P = Z \rho RT \tag{7}
\]

as long as a suitable method for calculating \( Z \) exists. For carbon dioxide and nitrous oxide, very accurate equations of state have been developed\(^8,11\) that can be used for this purpose. By taking the ratio of Eq. (5) to Eq. (4), a compressibility correction factor can be found:
Figure 28. Piping & instrumentation diagram.

\[ Y' = \sqrt{\frac{P}{2\Delta P}} \left( \frac{2n}{n-1} \right) \left( 1 - \frac{\Delta P}{P_T} \right)^{\frac{n}{2}} \left[ 1 - \left( 1 - \frac{\Delta P}{P_T} \right)^{\frac{n+1}{n}} \right] \] (8)

\[ Y = \sqrt{\left( \frac{P_C}{P_T} \right)^{\frac{\gamma}{\gamma-1}} \left( 1 - \frac{P_C}{P_T} \right)^{(\gamma-1)/\gamma}} \] (9)

Y’ generally lies between the incompressible value and the ideal gas compressibility correction factor.\(^{17}\)

Additionally, as \(\Delta P \to 0\) or \(Z \to 0\), \(Y' \to 1\), and as \(n \to \gamma\), \(Y' \to Y\).

To evaluate the impact of Eq. (8) on the measurements made in this work, \(Y'\) is plotted against reduced temperature in figure 29 for both saturated carbon dioxide and saturated nitrous oxide. The largest \(\Delta P\)
measurable with the experimental system utilized in this work is roughly $\Delta P_{\text{max}} = 0.35$ MPa, or 50 psi, and figure 29 is plotted with $\Delta P = \Delta P_{\text{max}}$. Also shown is the fluid compressibility factor defined in Eq. (7).

For the range of interest of this work it is clear that the compressibility correction factor, $Y'$, is very near 1.0 and therefore Eq. (4) is used without loss of accuracy.

![Figure 29. Compressibility correction for venturi flowmeters.](image)

References


